The Binomial Expansion Method Applied to CBO/CLO Analysis

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INTRODUCTION

Moody's ratings of collateralized bond obligations (CBOs) and collateralized loan obligations (CLOs) are ultimately based on the expected loss concept. Thus, the need for an accurate method of estimating the expected loss for the notes to be rated is of paramount importance.

A number of methods can be used to estimate the expected loss, ranging from Monte Carlo simulation techniques (which are fairly accurate but cumbersome to implement and computationally expensive to run) to rather simple single-event models (which are easy to implement but much less accurate).

An alternative to simulation or single-event models is the so-called Binomial Expansion Technique (BET), which combines the best of two worlds: a high degree of accuracy coupled with computational friendliness (in terms of both speed and implementation). This special report briefly describes the BET method in Moody's analysis of CBOs and CLOs.

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WHY USE THE BET?

The BET is a straightforward approach to estimating the total expected loss for a note in a CBO/CLO structure. It offers a number of advantages in determining the appropriate rating:

- It captures the effects of “tail events,” by accounting for all possible default scenarios.
- Its implementation is much less computationally intensive than that of a Monte Carlo simulation.
- It makes the so-called stability (or sensitivity) analysis which is required for highly rated investment grade notes fairly straightforward. (This topic will be covered in a future Special Report.)

THE BET METHOD

The BET method is based on the *diversity score concept.* The idea is to use the diversity score to build a hypothetical pool of *uncorrelated* and *homogeneous* assets (bonds or loans) that will mimic the default behavior of the original pool.

Let *D* be the diversity score of the collateral portfolio. Then, the behavior of the original pool can be modeled using a fictitious portfolio consisting of *D* bonds, each of which has the same par value (total collateral par value divided by *D*). It is also assumed that all these bonds have the same probability of default (determined by the weighted average probability of default of the original pool).

Finally, as far as defaults are concerned, the behavior of this homogeneous pool of *D* assets can be fully described in terms of *D* possible scenarios: one default, two defaults... up to *D* defaults. The probability *Pj* that scenario *j* (*j* defaults) could happen can be computed using the so-called binomial formula:

$$Pj = \frac{D!}{j!(D-j)!}\, p^{j}\,(1-p)^{D-j}$$

where *p* represents the weighted average probability of default of the pool (stressed by the appropriate factor).

Let *Ej* be the loss for the note to be rated under scenario j. (The loss, expressed as a percentage, can be easily computed by taking the present value of the cash flows received by the note holder, assuming there are j defaults, and using the note coupon as the discount factor).

Finally, the total expected loss, considering all possible default scenarios, is calculated as follows:

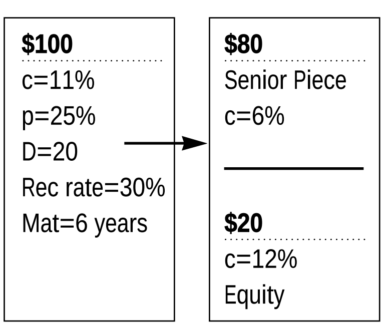
$$\text{Expected Loss} = \sum_{j=1}^{D} P_j E_j$$

AN EXAMPLE OF A BET APPLICATION

Consider the simple two-tier structure depicted in *Chart 1.* Assume that the collateral pool has a diversity score of 20, an average probability of default of 25% (after factoring in the stressing factor), a recovery rate of 30%, a six-year time to maturity, and pays an average coupon of 11%. Moreover, assume – just for simplicity – that all bonds are bullets, that there are no overcollateralization or interest rate triggers, and that the excess cash is reinvested at 11% per year. And obviously, the senior piece has priority to receive the cash flows from the collateral.

Chart 1

Hypothetical CBO Structure



In principle, the senior note is supposed to receive the following cash flows (on a semiannual basis): {2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4 + 80}. As long as the number of defaults remains below ten, the senior note will experience no losses. However, starting with ten defaults, the senior note will suffer increasing losses (see *Table 1*). The losses under each default scenario are computed simply by discounting, applying a 6% discount rate, whatever cash flows the senior note receives and comparing that present value with $80.

For example, if there are 10 defaults, the senior note receives {2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 2.4, 78.86} yielding a loss of 3.1%. In this case, it has been assumed that the defaults are front-loaded – that is, 50% occur at the end of year one and 10% at the end of each year for the five subsequent years.

Table 1 summarizes the results of the BET computation. The first column shows the number of defaults under the scenario; the second column shows the probability that that scenario will occur; and the third column shows the loss under that scenario.

The total expected loss is as follows:

$$\text{Expected Loss} = 0.3171\% \times 0.00\% + \ldots$$
$$\ldots + 0.00\% \times 45.1629\% = 0.067\%$$

Table 1
Summary of BET Calculation

# of Def.	Probability Scenario(%)	Loss(%)
0	0.3171	0.0000
1	2.1141	0.0000
2	6.6948	0.0000
3	13.3896	0.0000
4	18.9685	0.0000
5	20.2331	0.0000
6	16.8609	0.0000
7	11.2406	0.0000
8	6.0887	0.0000
9	2.7061	0.0000
10	0.9922	3.1026
11	0.3007	7.8958
12	0.0752	12.6890
13	0.0154	17.4822
14	0.0026	22.2754
15	0.0003	27.0686
16	0.0000	31.5621
17	0.0000	34.7819
18	0.0000	38.0531
19	0.0000	41.6080
20	0.0000	45.1629

Table 2 shows the expected loss for different ratings and different maturities based on Moody's idealized historic data. According to this table (see six-year column), the senior note would be rated **Aa3** (the cut-off value is 0.10065% for the **Aa3**).

It is also interesting to see the variation of the expected loss (and hence, the rating) as a function of the diversity, *D*, assuming all the remaining variables are kept constant. *Chart 2* depicts such a graph. Clearly, a variation in the value of *D* has a major impact for low-diversity pools; for higher values of *D*, the expected loss tends to be much more stable.

CONCLUSION

This example demonstrates the application of the BET to the analysis of a CLO/CBO. Of course, additional modeling complexities arise in real situations, which must address amortization, reinvestment criteria, overcollateralization tests, management fees, swaps, caps, different priority of payments, and the like. Also, more nonhomogeneous portfolios (for example, portfolios in which a few bonds account for a large portion of the collateral portfolio) might require some special modifications of the BET method. These extreme cases must be examined carefully.

Chart 2
Expected Loss versus Diversity

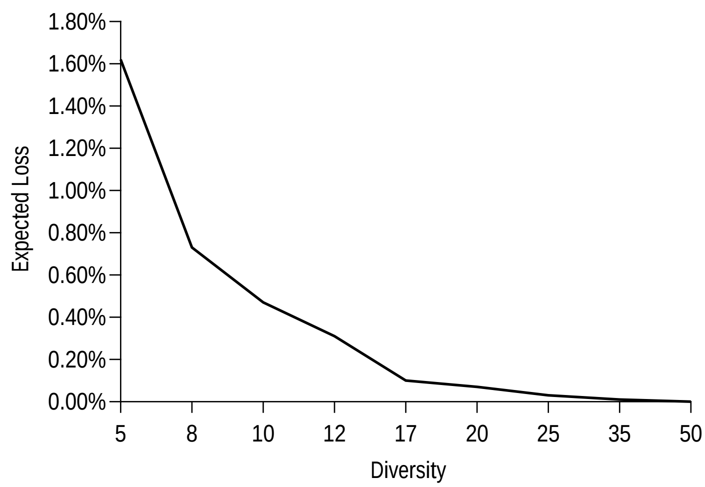


Table 2

Moody's "Idealized" Cumulative Expected Loss Rates (%)

Rating	Year 1	2	3	4	5	6	7	8	9	10
Aaa	0.000028	0.00011	0.00039	0.00099	0.00160	0.00220	0.00286	0.00363	0.00451	0.00550
Aa1	0.000314	0.00165	0.00550	0.01155	0.01705	0.02310	0.02970	0.03685	0.04510	0.05500
Aa2	0.000748	0.00440	0.01430	0.02585	0.03740	0.04895	0.06105	0.07425	0.09020	0.11000
Aa3	0.001661	0.01045	0.03245	0.05555	0.07810	0.10065	0.12485	0.14960	0.17985	0.22000
A1	0.003196	0.02035	0.06435	0.10395	0.14355	0.18150	0.22330	0.26400	0.31515	0.38500
A2	0.005979	0.03850	0.12210	0.18975	0.25685	0.32065	0.39050	0.45595	0.54010	0.66000
A3	0.021368	0.08250	0.19800	0.29700	0.40150	0.50050	0.61050	0.71500	0.83600	0.99000
Baa1	0.049500	0.15400	0.30800	0.45650	0.60500	0.75350	0.91850	1.08350	1.24850	1.43000
Baa2	0.093500	0.25850	0.45650	0.66000	0.86900	1.08350	1.32550	1.56750	1.78200	1.98000
Baa3	0.231000	0.57750	0.94050	1.30900	1.67750	2.03500	2.38150	2.73350	3.06350	3.35500
Ba1	0.478500	1.11100	1.72150	2.31000	2.90400	3.43750	3.88300	4.33950	4.77950	5.17000
Ba2	0.858000	1.90850	2.84900	3.74000	4.62550	5.37350	5.88500	6.41300	6.95750	7.42500
Ba3	1.545500	3.03050	4.32850	5.38450	6.52300	7.41950	8.04100	8.64050	9.19050	9.71300
B1	2.574000	4.60900	6.36900	7.61750	8.86600	9.83950	10.52150	11.12650	11.68200	12.21000
B2	3.938000	6.41850	8.55250	9.97150	11.39050	12.45750	13.20550	13.83250	14.42100	14.96000
B3	6.391000	9.13550	11.56650	13.22200	14.87750	16.06000	17.05000	17.91900	18.57900	19.19500
Caa	14.300000	17.87500	21.45000	24.13400	26.81250	28.60000	30.38750	32.17500	33.96250	35.75000